Analysis of Torsional Vibration in Elliptical Gears

Kazuteru NAGAMURA*1 and Kiyotaka IKEJO*2

*1 Department of Mechanical System Engineering, Hiroshima University
1- 4- 1, Kagamiyama, Higashi- Hiroshima, 739-8527, JAPAN
nagamura @ mec.hiroshima-u.ac.jp

*2 Department of Mechanical System Engineering, Hiroshima University
1- 4- 1, Kagamiyama, Higashi- Hiroshima, 739-8527, JAPAN
ikejo @ mec.hiroshima-u.ac.jp

Abstract

An elliptical gear pair is a mechanism which transmits a non-uniform rotation as efficiently as a cam. Because of the non-uniform rotation of non-circular gears, the torque varies significantly, causing an increase of vibration and noise. In this study, we investigated the vibration characteristics of two elliptical gear pairs: a single elliptical gear pair and a double elliptical gear pair. The torque variation of the shafts, the tooth root stress of the gear, the angular motion of the shafts, and the circumferential vibration acceleration of the test gear were measured in a running test. Furthermore, a program to calculate the vibration of the testing machine with the elliptical gear pair was also developed, and calculated the torque variation of the shafts, and the circumferential vibration acceleration of the test gear. Then, the results calculated by the program and the experimental results were compared to confirm the validity of the calculation program.

Keywords: elliptical gear, vibration, non-uniform rotation, torque variation

1 Introduction

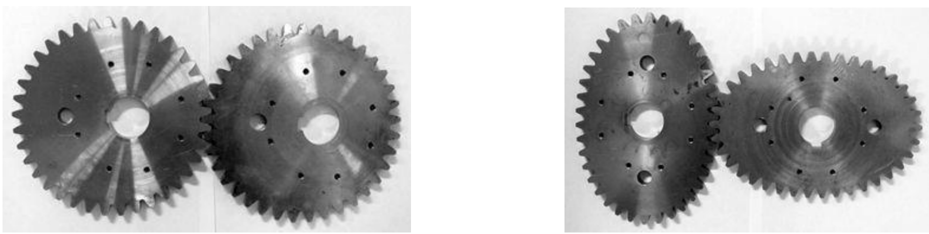
An elliptical gear drive is a typical non-circular gear drive. It can transmit variable-ratio rotation and power simultaneously, as efficiently as a cam. An elliptical gear drive has such advantages as its simplicity and small size, its ability to transmit a heavy load, its high durability, and its low friction loss in comparison with a cam, because of the low level of sliding on its contact surface [1]. However, because of its shape and non-uniform rotation it has torque variation, and the vibration and noise of elliptical gear drives are larger than those of circular gear drives which rotate uniformly. Therefore, elliptical gear drives are considered unsuitable for operating at high speeds. Consequently, elliptical gear drives are used at low speeds where negligible vibration is generated. There are some studies [1], [2] on the design of elliptical gear drives, but few studies that refer to the vibration of elliptical gear drives during operation. In this study, we investigated the vibration characteristics of two elliptical gear pairs: a single elliptical gear pair and a double elliptical gear pair. In the running test, we measured the tooth root stress and the circumferential vibration acceleration of the test gear pairs as well as the torque variation and the angular rotation speed of driving and driven shafts to investigate the vibration characteristics of the elliptical gear pairs.

Furthermore, we established a torsional vibration model of the gear drive systems with elliptical gear pair. The vibration model took account of the particular gear mesh characteristics of the elliptical gear. On the basis of the vibration model, a program to calculate the vibration of the testing machine with the elliptical gear pair was developed. By using the calculation program, we calculated the torque variation of the shafts, and the circumferential vibration acceleration of the test gear pairs. The results calculated by the program and the experimental results were compared to confirm the validity of the program.

2 Experimental procedure and results

2.1 Elliptical gears

In this study, the vibration characteristics of a single elliptical gear pair and a double elliptical gear pair are investigated by comparing them with two circular gear



(a) Single elliptical gear pair **(b) Double elliptical gear pair**

Fig. 1 Elliptical gear pairs

Table 1 Specification of test gears

Gear	Single elliptical gear	Circular gear I	Double elliptical gear	Circular gear II
Module m [mm]	4			
Standard pressure angle α_0 [deg]	20			
Number of teeth z_1/z_2	37/37		38/38	
Center distance a [mm]	149.51		149.1	
Face width b [mm]	10			
Pitch circle radius r	−	74	−	76
Maximum and minimum pitch curve radius r_{max}/r_{min} [mm]	89.68/59.79	−	89.44/59.63	−
Addendum modification coefficient x	−	0.14	−	−0.39
Contact ratio ε	−	1.68	−	1.96
Method of finishing teeth	Pinion cutter	Hobbed	Pinion cutter	Hobbed
Material	JIS G4051 S45C Thermal refining steel			

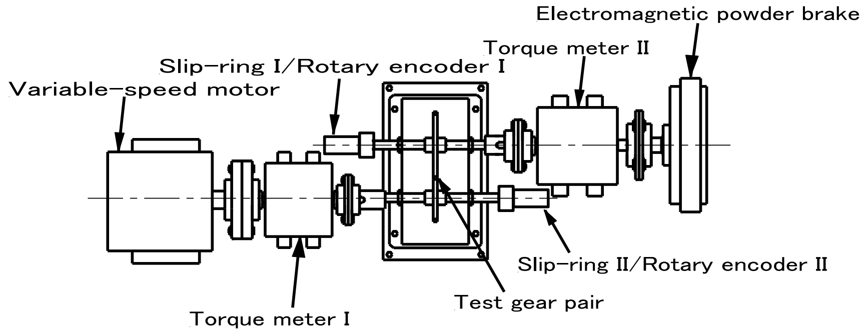


Fig. 2 A scheme of power absorption type gear testing machine

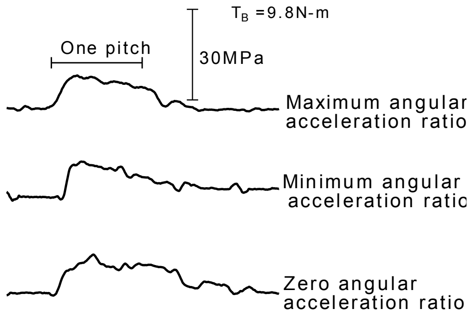
pairs which have the same module, pressure angle, number of teeth, center distance, face width and material as the two elliptical gear pairs, respectively. The driving and driven gears of the test gear pairs have the same gear geometry as each other. The single elliptical gear pair shows one variation of the angular velocity and the acceleration in one gear rotation. The double elliptical gear pair presents two variations of the angular velocity and the acceleration in one gear rotation. **Figure 1** shows two elliptical gear pairs. The test gears are specified in Table 1. As shown in **Table 1**, the circular gears I and II correspond to the single elliptical gear and the double elliptical gear, respectively.

2.2 Experimental procedure

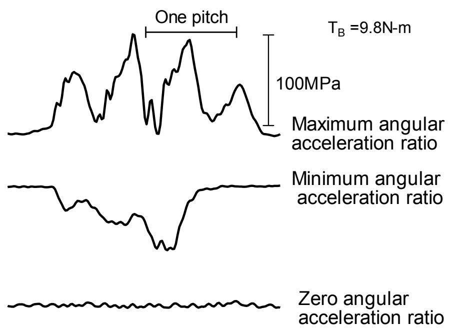
The power absorption type gear testing machine shown in **Fig. 2** was used for the experiments. The testing machine consisted of a variable-speed motor for driving, two torque meters, two slip-rings (or two rotary encoders), a test gear pair, and an electromagnetic powder brake for loading. The tooth root stress, the circumferential vibration acceleration of the test gears and the torque variation, and the angular rotation speed of driving and driven shafts were measured in the running test. The load torque T_B was set at 9.8 N-m or 19.6 N-m by the electromagnetic powder brake. The experiment was carried out at the driving gear speed n_1 of 120 to 600 rpm.

2.3 Experimental results

Figure 3 shows the tooth root stress waveform of the driven gear measured by the strain gauge bonded on the tooth root fillet as an example. The tooth root stress for elliptical gears was measured at three teeth, where the angular acceleration ratio have the maximum value, the

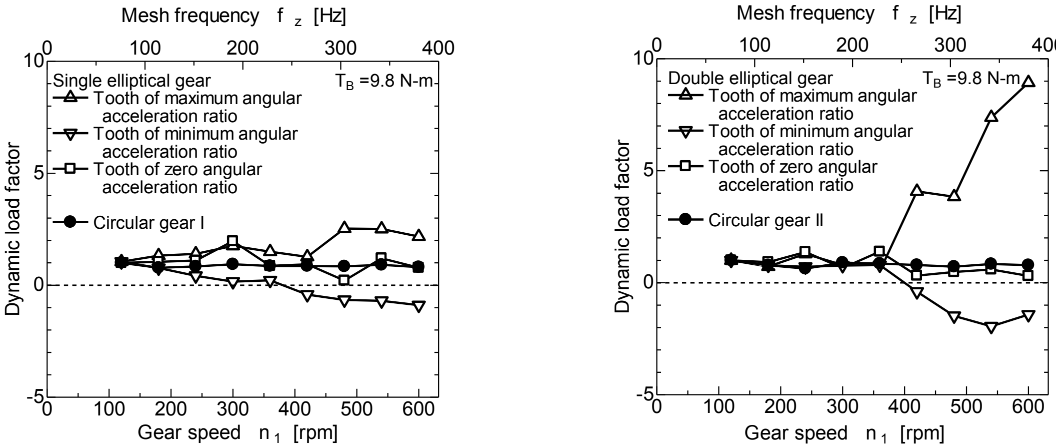


(a) n_1=120 rpm



(b) n_1=600 rpm

Fig. 3 Tooth root stress of double elliptical gear (T_B =9.8N-m)



(a) Single elliptical gear **(b) Double elliptical gear**

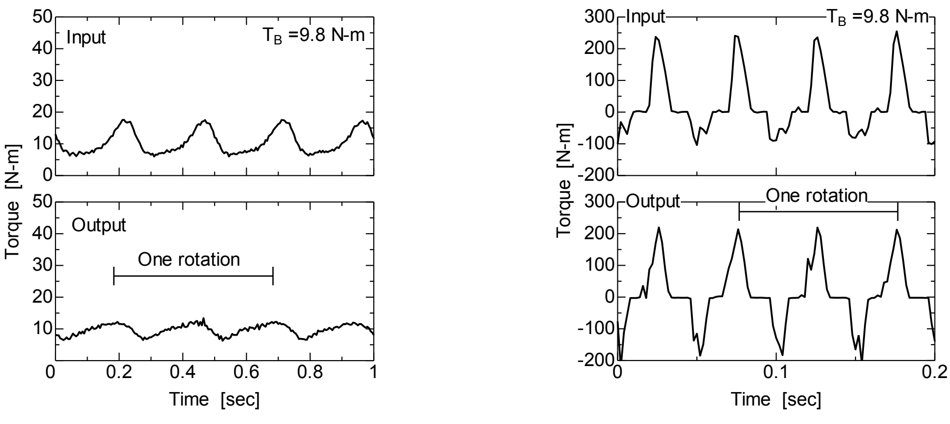
Fig. 4 Dynamic load factor (T_B =9.8N-m)

minimum one and zero. At the driving gear speed of 120 rpm, root stresses of three teeth are about the same. However, at 600 rpm tooth root stresses are strikingly different in that the root stress for the tooth of the maximum angular acceleration ratio presents a resonance, for the tooth of the minimum angular acceleration ratio the non-working flanks, which are not usually in contact, collide with each other, and for the tooth of the zero angular acceleration ratio there is tooth separation in contacting teeth.

Figure 4 presents a speed sweep of the dynamic load factor of elliptical gears. The dynamic load factor is defined as the ratio of the maximum tooth root stress at each gear speed to the maximum tooth root stress at 120 rpm, which is considered to be a static stress. For the circular gears, the dynamic load factor has **a** little fluctuation, and its value approaches unity. For the elliptical gears, the dynamic load factor changes depending on the tooth. The dynamic load factor at the tooth with the maximum angular acceleration ratio increases as the gear speed increases, while the dynamic load factor at the tooth with the minimum angular acceleration ratio decreases with the increase of gear speed. This is because the vibration is significantly larger on their tooth because of the large inertial torque. In addition, the dynamic load factor at the tooth with the minimum angular acceleration ratio becomes negative over 400 rpm. This indicates that there is tooth separation in contacting teeth, and that the non-working flanks collide with each other.

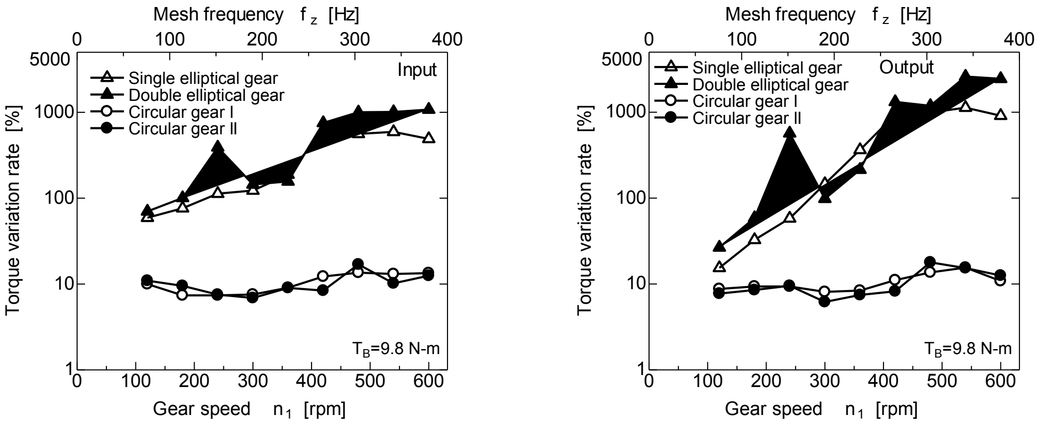
Figure 5 shows the waveform of input and output torques for the double elliptical gear pair measured with the torque meters I and II. At 120 rpm, the input torque varies according to the angular velocity ratio curves of elliptical gear. This is because the inertial torque of the elliptical gear is still small and the input torque is mainly determined by the load torque and the varying angular velocity ratio. With the increase of gear speed, the variations of input torque become significantly larger. It is thought that the inertial torque of the driven side caused by the non-uniform rotation of elliptical gears becomes larger with the increase of gear speed.

Figure 6 shows the torque variation ratios versus the gear speeds for the four test gears. The torque variation ratio is defined as the ratio of the amplitude of torque to the average torque value which is the load torque in this



(a) n_1=120 rpm **(b) n_1=600 rpm**

Fig. 5 Torque waveform for double elliptical gear (T_B =9.8N-m)



(a) Input torque **(b) Output torque**

Fig. 6 Torque variation ratio (T_B =9.8N-m)

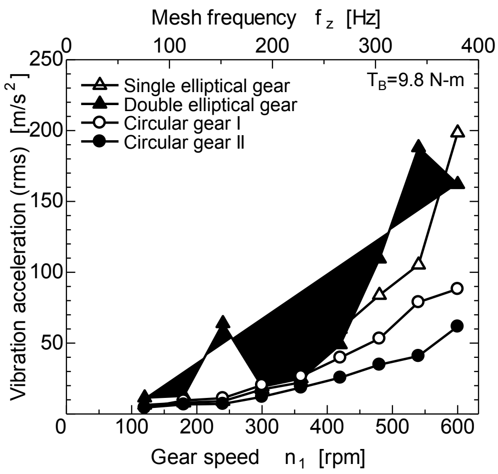


Fig. 7 Circumferential vibration acceleration (T_B =9.8N-m)

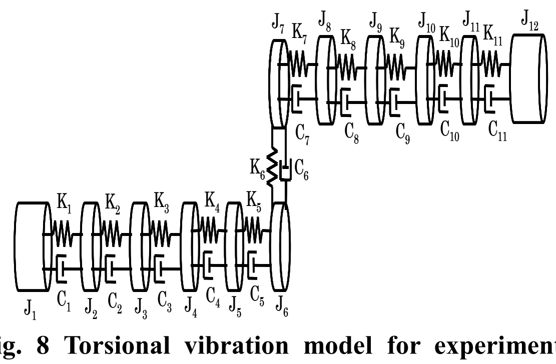


Fig. 8 Torsional vibration model for experimental apparatus

study. As shown in **Fig. 6**, the torque variation ratios of elliptical gears are much larger than those of circular gears, and increase rapidly over a certain rotation speed where the tooth separation is likely to occur.

Figure 7 shows the root mean square value of the circumferential vibration acceleration of the test gears. Two accelerometers were employed for measuring the circumferential vibration acceleration of the test gears, and the two accelerometers were attached to the driving gear in an axial symmetrical position with each other. The circumferential vibration acceleration can be obtained by averaging the output signals from the two accelerometers. The root mean square value of the circumferential vibration acceleration of the elliptical gear pairs increases rapidly and is considerably larger than that of the corresponding circular gears over 400 rpm. This is due to the torque and the gear speed changing drastically because elliptical gears have an elliptical pitch curve.

3 Vibration analysis

3.1 Vibration model

In order to investigate the vibration characteristics of elliptical gears, we established a torsional vibration model for the experimental apparatus shown in **Fig. 2**. **Figure 8** illustrates a torsional vibration model of the power absorption type gear testing machine. The elements which are of larger moment of inertia and smaller stiffness are considered as masses, while those which are of smaller moment of inertia and larger stiffness are considered as springs and dampers. The torsional vibration equation for the gear system are expressed as follows;

$$\left.\begin{aligned}
&J_1\ddot{\theta}_1+C_1(\dot{\theta}_1-\dot{\theta}_2)+K_1(\theta_1-\theta_2)=T_M\\
&J_2\ddot{\theta}_2+C_1(\dot{\theta}_2-\dot{\theta}_1)+K_1(\theta_2-\theta_1)+C_2(\dot{\theta}_2-\dot{\theta}_3)+K_2(\theta_2-\theta_3)=0\\
&J_3\ddot{\theta}_3+C_2(\dot{\theta}_3-\dot{\theta}_2)+K_2(\theta_3-\theta_2)+C_3(\dot{\theta}_3-\dot{\theta}_4)+K_3(\theta_3-\theta_4)=0\\
&J_4\ddot{\theta}_4+C_3(\dot{\theta}_4-\dot{\theta}_3)+K_3(\theta_4-\theta_3)+C_4(\dot{\theta}_4-\dot{\theta}_5)+K_4(\theta_4-\theta_5)=0\\
&J_5\ddot{\theta}_5+C_4(\dot{\theta}_5-\dot{\theta}_4)+K_4(\theta_5-\theta_4)+C_5(\dot{\theta}_5-\dot{\theta}_6)+K_5(\theta_5-\theta_6)=0\\
&J_6\ddot{\theta}_6+C_5(\dot{\theta}_6-\dot{\theta}_5)+K_5(\theta_6-\theta_5)+C_6r_{b6}(r_{b6}\dot{\theta}_6-r_{b7}\dot{\theta}_7)+K_6r_{b6}(r_{b6}\theta_6-r_{b7}\theta_7)=0\\
&J_7\ddot{\theta}_7+C_6r_{b7}(r_{b7}\dot{\theta}_7-r_{b6}\dot{\theta}_6)+K_6r_{b7}(r_{b7}\theta_7-r_{b6}\theta_6)+C_7(\dot{\theta}_7-\dot{\theta}_8)+K_7(\theta_7-\theta_8)=0\\
&J_8\ddot{\theta}_8+C_7(\dot{\theta}_8-\dot{\theta}_7)+K_7(\theta_8-\theta_7)+C_8(\dot{\theta}_8-\dot{\theta}_9)+K_8(\theta_8-\theta_9)=0\\
&J_9\ddot{\theta}_9+C_8(\dot{\theta}_9-\dot{\theta}_8)+K_8(\theta_9-\theta_8)+C_9(\dot{\theta}_9-\dot{\theta}_{10})+K_9(\theta_9-\theta_{10})=0\\
&J_{10}\ddot{\theta}_{10}+C_9(\dot{\theta}_{10}-\dot{\theta}_9)+K_9(\theta_{10}-\theta_9)+C_{10}(\dot{\theta}_{10}-\dot{\theta}_{11})+K_{10}(\theta_{10}-\theta_{11})=0\\
&J_{11}\ddot{\theta}_{11}+C_{10}(\dot{\theta}_{11}-\dot{\theta}_{10})+K_{10}(\theta_{11}-\theta_{10})+C_{11}(\dot{\theta}_{11}-\dot{\theta}_{12})+K_{11}(\theta_{11}-\theta_{12})=0\\
&J_{12}\ddot{\theta}_{12}+C_{11}(\dot{\theta}_{12}-\dot{\theta}_{11})+K_{11}(\theta_{12}-\theta_{11})=-T_B
\end{aligned}\right\} \cdots(1)$$

where, index i is the element number, J_i is the moment of inertia, θ_i is the angular displacement, T_M is the torque of the motor, T_B is the set-up torque of the electromagnetic powder brake, C_i is the damping coefficient of damper i, and K_i is spring constant of the stiffness i, r_{b6} and r_{b7} are the base circle radius of driving gear and driven gear respectively. For the elliptical gears, the base circle radii r_{b6} and r_{b7} change depending on the tooth.

In this study, the differential equation (1) was solved by fourth order Runge-Kutta-Gill method employing the parameter shown in **Table 2**. The torque in the torque meters and the circumferential vibration acceleration of the test gears were calculated.

Table 2 Parameter for Vibration model

Elements number	Stiffness K [N-m/rad]
1	197957
2	2421
3	4762
4	157000
5	59736
6	178249358
7	59736
8	157000
9	4762
10	1852
11	197957

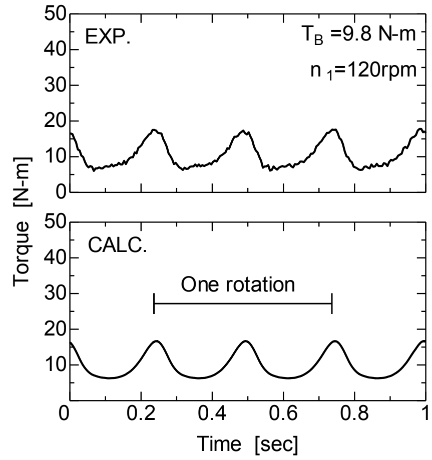
Elements number	Moment of inertia J [kg-m2]
1	0.10000
2	0.00387
3	0.00387
4	0.00065
5	0.00065
6	0.00417
7	0.00417
8	0.00065
9	0.00065
10	0.00206
11	0.00206
12	0.00950

3.2 Results and discussion

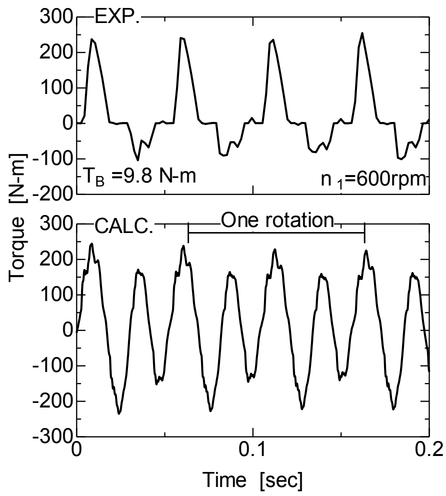
Figure 9 compares the calculated results and experimental results of the input torque waveforms for the double elliptical gear. From the comparison of the calculated results and experimental results shown in Fig. 9, it can be found that the calculated results almost agree with the experimental results. However, at 600 rpm the input torque occasionally has zero or rather negative value in the experiment. This is because the tooth separation and the collision of non-working flank were not successfully applied to the calculation.

Figure 10 shows the calculated results of the input torque variation ratios for the four test gear drives. The calculated results of torque variation ratios of elliptical gear drives almost agree with the experimental results.

Figure 11 compares the circumferential vibration acceleration between the calculated results and the experimental results. The calculated values agree with the experimental results.



(a) n_1=120 rpm



(b) n_1=600 rpm

Fig. 9 Comparison between experimental and calculated results of input torque waveform for double elliptical gear (T_B =9.8N-m)

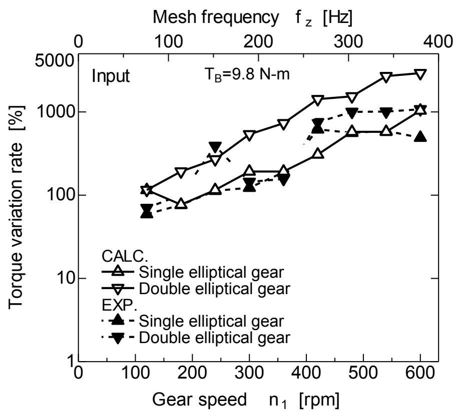
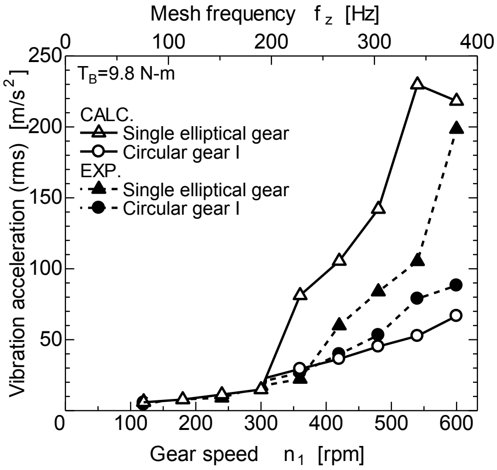
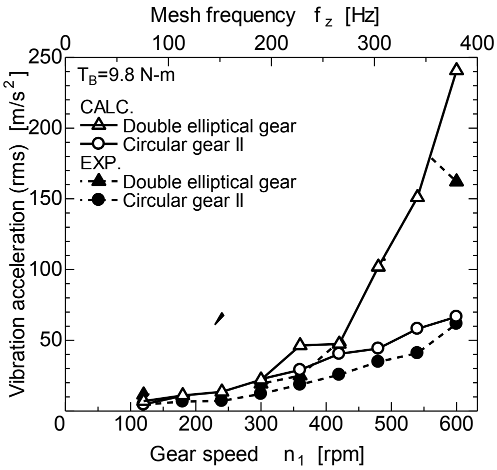


Fig. 10 Calculated results of the input torque variation ratios (T_B =9.8N-m)



(a) Single elliptical gear



(b) Double elliptical gear

Fig. 11 Calculated results of circumferential vibration acceleration (T_B =9.8N-m)

4 Conclusions

The results in this study may be summarized as follows:

1. The elliptical gears in this study have significantly large dynamic load, and are likely to provide tooth separation at lower gear speed than circular gears.
2. The elliptical gears have larger vibration than the circular gears except for at very low gear speeds.
3. A method to calculate the torque variation of shaft and the circumferential vibration acceleration for the elliptical gear is presented. The calculated results by the present method agree with the experimental results.

References

[1] H. KATORI, "Hienkei-Haguruma no Sekkei, Seisaku to Ouyou (Design, Manufacture and Application of Non-circular Gears)", Nikkan Kogyo Shinbun, Ltd., pp.7-55, 2001, (in Japanese).

[2] F. L. Litvin, A. Fuentes, I. Gonzalez, and K. Hayasaka, "Non-circular Gears Design and Generation", CAMBRIDGE UNIVERSITY PRESS, pp.71-92, 2009.

Received on December 30, 2013
Accepted on February 10, 2014