# Design and Manufacture of New Circular-Arc Tooth-Trace Gears
## (Design of Rack and Pinion based on Transverse System)

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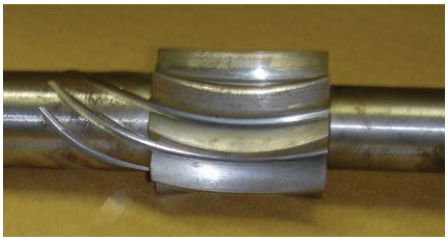
## Abstract
In general, the circular-arc tooth-trace (CATT) gears are considered as a special type of spiral bevel gears whose shaft-angle is zero. The CATT gears were manufactured using the conventional circular face-mill cutters. In this paper, the new CATT gears have been developed using 3D-CAD/CAM; the tooth profile of pinion gears has the same involute curve at the any section through the face-width. Similarly the tooth profile of the CATT's rack is a straight line at the both sides of the face-width.
**Keywords:** design, manufacture, gear, rack & pinion, 3D-CAD/CAM, CNC machine tool

## 1 Introduction
**Figure 1** shows the circular-arc tooth-trace (CATT) gear, a face-mill cutter and a cup-shaped grinding wheel to manufacture the gear. The circular-arc tooth-trace gear features have excellent characteristics such as being free from axis thrust, hard to get in contact with one side (end tooth bearing) by means of the action of self-centering, capable of machining in exceedingly high accuracy provided that cutting and grinding are made by a single machining tool [1], [2]. However these gears have not been spread because a special machine and a special tool are needed to manufacture the gears [3]~[6].

In this study, the new CATT gears were manufactured in such a manner as follows instead of using the special machine with the circular face-mill cutter. At first, a solid model of the new CATT gear was made by means of 3D-CAD using the calculation of standard coordinates of involute tooth profile or tooth-trace. And then the new CATT gear was manufactured by a CNC milling machine and a ball end-mill. The new CATT gear has a standard involute curve at any cross section of the tooth face-width. Hence the new CATT gear differs from the conventional CATT's tooth curve. The new CATT gears have a possibility to be spread in many fields because the new CATT rack has a linear tooth curve at any cross section.

The features, the design method and the way manufacturing the new CATT gear will be explained in the paper.



**(a) Pinion with circular-arc tooth-trace**

**(b) Circular face-mill cutter**

**(c) Cup-shaped grinding wheel**

**Fig. 1 Circular-arc tooth-trace gear and typical tool**

## 2 Features of the CATT's Gears
In general, arising from its circular-arc tooth-trace, the preferable features in the use of and manufacturing of the gears are pointed out below:

[GP-1] Crowning (convexity) can very easily be attached. When concave and convex surfaces are shaped by separate curvature radius, the convexity equivalent to the difference of the radii can be attached.

[GP-2] Hardly being contact at the end of tooth bearing. When freedom is given to one of the axis in an axial direction, the gear can automatically move in an axial direction, and attaching of the gear moves to the region around the center.

[GP-3] Cutting and grinding can be made with the same exclusive machine. When grinding is made by attaching a cup-shaped grinding wheel instead of a circular face-mill cutter, the work can be made with high accuracy.

On the other hand, the bad points of the circular-arc tooth-trace gear are designated as shown below.

[BP-1] Plural numbers of gear materials cannot be cut by a single tool.

[BP-2] When the distance between the pivotal points of both the gears is fixed, the gear element cannot be slid into the gear from side (building-up limitation).

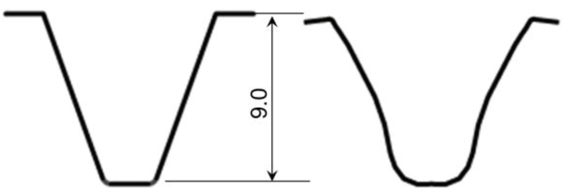
[BP-3] Even with the same module, engagement can be exclusively made with the same curvature radii of the tooth-trace (a problem of interchange ability)

## 3 Specification of the CATT gears

A set of a pinion and a rack was manufactured for a design trial. The principal dimensions of the circular-arc tooth-trace gears are listed in **Table 1**.

### 3.1 Design dimensions of the CATT rack & pinion

For the gears to be tentatively made in this study, the involute-type standard gear profile with module 4 and pressure angle 20° is used. The tooth face-width is 50 mm, and the curvature radii of the tooth-trace are equally 70 mm both with concave and convex ones.




(a) Rack (b) Pinion

**Fig. 2 Tooth profile of CATT (Rack & pinion)**

**Table 1 Specification of the new CATT gears**

| Item | | Pinion | Rack |
|---|---|---|---|
| Module | m | 4 | |
| Number of teeth | z | 24 | - |
| Pressure angle | α | 20° | |
| Pitch circle dia. | dp | 96 | - |
| Base circle dia. | dg | 90.210 | - |
| Outside dia. | dk | 104 | - |
| Dedendum | ha | 4 | |
| Addendum | hr | 5 | |
| Face-width | b | 50 | |
| Radius of curvature | R | 70 | |

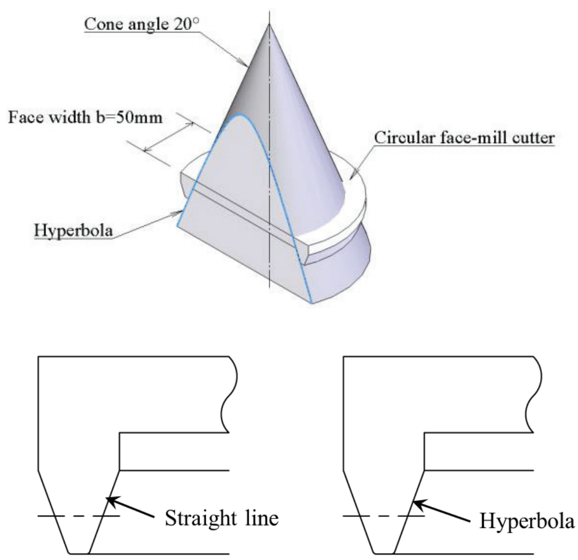
### 3.2 Tooth profile of the CATT gears

The standard tooth profile of this study is of the standard tooth profile with the module 4 throughout the whole cross section in a direction of the tooth face-width as illustrated in **Fig. 2**. The coordinate representing this tooth shape was geometrically calculated on a computer. To be concrete with the matter, an involute curve stretching from a base circle is at first calculated. With the tooth-root shape on the fundamental circle, interpolation was made using an adequate ellipse.

With the machining by means of the conventional cutter (**Figure 1(b)**) or cup-shaped grinding stone, a standard tooth profile is taken on the tooth width central part of the work. In the vicinity of the tooth width terminals, however, a tooth shape is slightly different from the profile at the mid-width section. Depicted in **Fig. 3** is a model of the circular face-mill cutter by which a circular-arc tooth-trace rack of pressure angle 20° is cut with the work width 50 mm. This becomes a cutting-edge shape (part of hyperbola) for cutting the both sides of the face-width. In the design condition in this study, the shape yields a difference of approximately 6 μm from the cut edge shape (cutter profile error) on the tooth width center.

## 4 Design of the CATT gears using 3D-CAD

In this study, a 3 dimensional solid model was manufactured using mainly 3D-CAD SolidWorks 2012. In the design process, the circular-arc tooth-traces of a rack and pinion (standard gear) shown in **Fig. 2** were both generated by the sweep-cut with an approximate tooth profile and a circular arc as a sweep guide numerically derived from an equation. Therefore the circular-arc tooth-trace gears (rack & pinion) designed and manufactured in this study are quite the same on both face sides throughout the face-width. Principal design dimensions are shown in **Table 1**. An example of




**(a) At center** **(b) At both side**

**Fig. 3 Circular face-mill cutter and tooth profile error on CATT's rack**

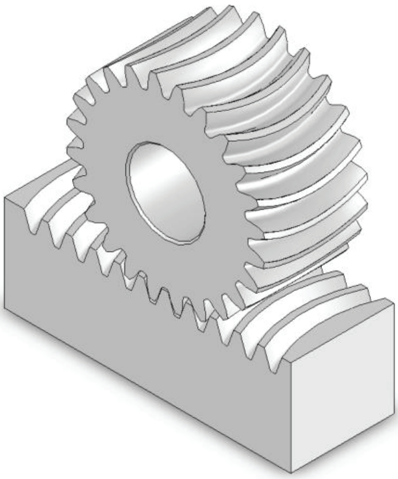


**Fig. 4 3D-CAD solid model of rack & pinion (m4, Z=24, R=70 mm)**

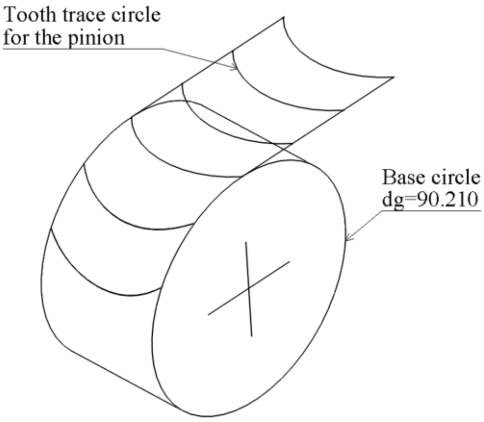



**Fig. 5 Illustration of curvature on plane of action**

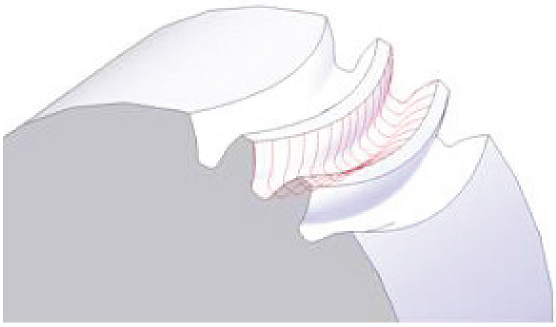


**Fig. 6 Tooth profile of CATT's gear**

the solid model of the complete circular-arc tooth-trace gears (rack & pinion) is illustrated in **Fig. 4**.

**4.1 Design of the CATT's rack**

The circular-arc tooth-trace, which is designed to be an arc of curvature radius 70 mm in a cross section in a normal direction on the pitch point of the tooth face-width, becomes an ellipse of the major axis 74.5 mm and the minor axis 70.0 mm on the pitch surface. Accordingly the cylinder having a 70 mm central line that crosses with the said normal at a right angle comes to be tightly contact with the rack along the tooth face.

**4.2 Design of the CATT's pinion gear**

Likewise with the case of the rack, both the left and right tooth surfaces are manufactured in the SWEPT CUT (3D-CAD command) holding in parallel the tooth profile of a standard involute curve (**Fig. 2(b)**). The guide curve to allow the pinion to sweep was manufactured based on a 3-dimensional curve that is made when an arc-trace (curvature: R = 70 mm) is depicted and wound onto the basic cylinder (**Fig. 5**). To be concrete with this, a standard curve formed by 51 points obtained at 1 mm width in a tooth face-width direction was dealt with as a guide curve for the swept cut. For the verification of the precision in modeling, the circular-arc tooth-trace gear was 3D-modeled by use of loft command with the 103 sections spacing with the interval of 0.5mm throughout the tooth width illustrated in **Fig. 6**.

## 5 Gear Cutting

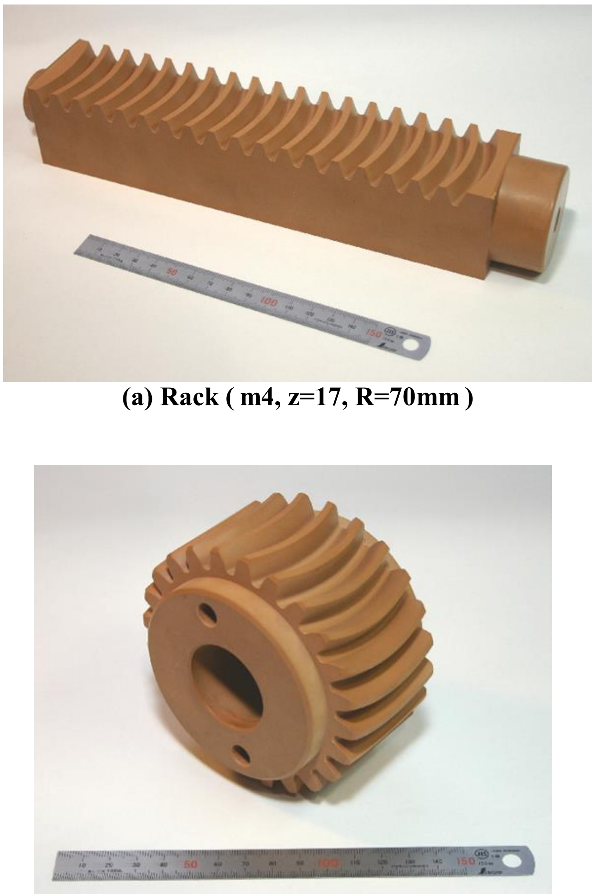
**5.1 CNC machine tool for cutting**

The circular-arc tooth-trace gears manufactured in this study is designed by using 3D-CAD so that they

**Fig. 7 CNC machine tool (Roland MDX-540)**

**Table 2 Cutting conditions for CNC-Machine tool**

| Item | | Pinion | Rack |
|---|---|---|---|
| End mill | | Ball-nose | |
| Tool diameter (mm) | | 3 | |
| Feed (m/min) | Roughing | 2.16 | 1.62 |
| | Finishing | 1.08 | 1.08 |
| Spindle speed (rpm) | Roughing | 10,000 | |
| | Finishing | 10,000 | |



**(a) Rack ( m4, z=17, R=70mm )**

**(b) Pinion ( m4, z=24, R=70mm )**

**Fig. 8 The circular-arc tooth-trace rack and pinion manufactured by MDX-540**

will form a standard involute tooth-profile throughout the whole cross section in the work width direction, and therefore it was impossible to manufacture the gears with the use of the manufacturing machines settled in the authors' university. The manufacture was made by using the CNC machine tool settled in Kyushu University (photographed in **Fig. 7**). Therefore as the blank material, relatively soft plastics such as Polyacetal or Cycowood, etc. was used from their preferable machinability.

### 5.2 Cutting condition

Cutting for the manufacture was made by a ball-end mill with a diameter 3 mm by separating the process into 2 steps of a rough cutting and a final finishing. The principal conditions in case of the cutting are listed in **Table 2**. It took approximately 8 hours to finish the pinion from the blank under the condition of spindle rotation speed 10,000 rpm and feed suggested to the material. The circular-arc tooth-trace gears (rack and pinion) are photographed in **Figure 8**. The final tooth face was considerably fine from a visual observation, and its gearing was also smooth. Accuracy of the tooth form, tooth-trace, etc. is not yet measured, and is now in consideration.

## 6 Conclusions

The new circular-arc tooth-trace gears were designed and manufactured according to the proposal made in this paper. As a result, the following were explained:

(1) The proposal was made with a method to relatively easily allow the circular-arc tooth-trace gears to be designed and manufactured with the use of 3D-CAD/CAM instead of the conventional circular-arc tooth-trace gears manufactured by a circular face-mill cutter.

(2) With the proposed method in this study using 3D-CAD/CAM to design and manufacture the new circular-arc tooth-trace gears, the tooth profiles are the same and the correct involute was obtained in any cross section throughout the tooth width, improving the degradation of the tooth profile near the tooth-ends caused by the geometrical constraint of the conventional circular face-mill cutter.

(3) It is ascertained from the smooth engagement that the CNC machine tool can generate a sufficient surface quality to the trial circular-arc tooth-trace gears.

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